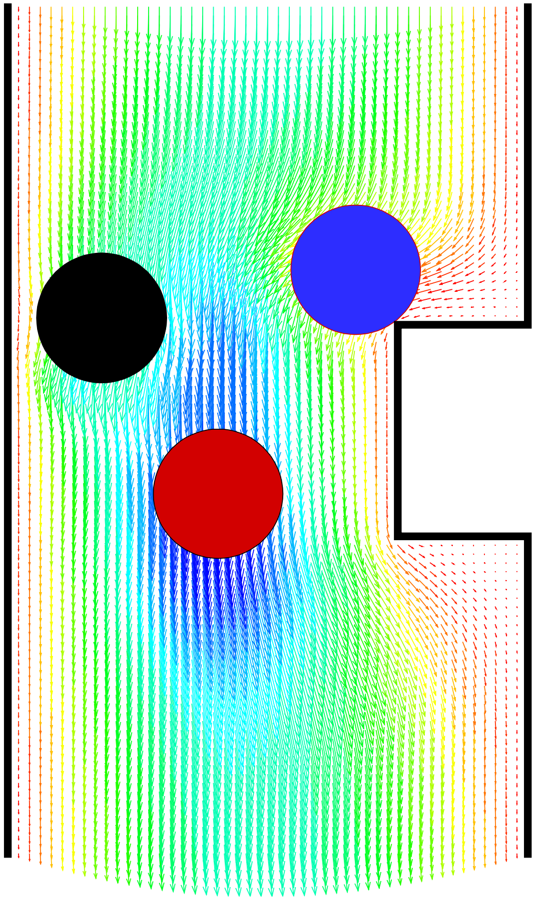
A Fast Algorithm for Particulate Micro-Flows in Complex Geometries



Particles in Poiseuille flow

Force-Coupling Method *(* Maxey)*

- Incorporates two-way coupling
- Gaussian distribution to represent forces on the fluid from the particles
- Particle radius

$$R = \sigma\sqrt{\pi}$$

$$\boxed{\Delta(x-Y) = \frac{1}{(2\pi\sigma^2)^{3/2}}\exp\left(\frac{(x-Y)^2}{2\sigma^2}\right)}$$

$$\rho\left(\frac{\partial u}{\partial t} + u \bullet \nabla u\right) = -\nabla p + \mu\nabla^2 u + \sum_n F_i^n \Delta(x-Y) \quad + \quad F_{ij}^n \frac{\partial\Theta(x-Y)}{\partial x_j}$$

Force Monopole for Fluid-Particle & Particle-Particle Interactions

Force Dipole for Wall-Flow & Wall-Particle Interactions

Particle Phase Motion

Particle position **Y**(t), velocity **V**(t) and angular velocity Ω in terms of vorticity ω

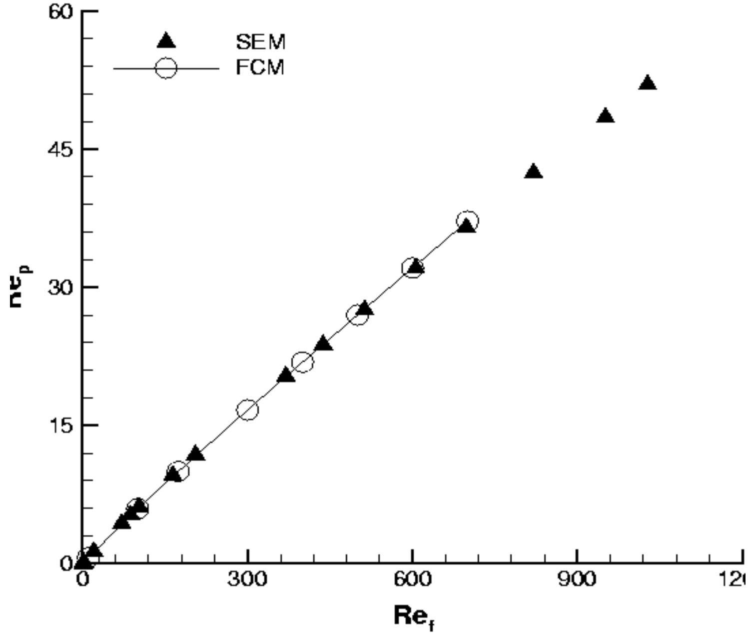
$$\frac{d\mathbf{Y}}{dt} = \mathbf{V}(t) = \int \mathbf{u}(\mathbf{x}, t)\Delta(\mathbf{x} - \mathbf{Y}(t))d^3\mathbf{x}$$

$$\Omega_i(t) = \frac{1}{2}\int \omega_i(\mathbf{x}, t)\Theta(\mathbf{x} - \mathbf{Y}(t))d^3\mathbf{x}$$

Force coefficient $\mathbf{F} = (m_P - m_F)(\mathbf{g} - \frac{d\mathbf{V}}{dt}) + \mathbf{F}_{\mathrm{CONTACT}}$

Force dipole G_{ij} set by net external torque and by moment of inertia, to maintain zero volume-averaged rate of strain inside volume occupied by particles

Comparison between SEM and FCM (Particles in a Periodic Lattice)

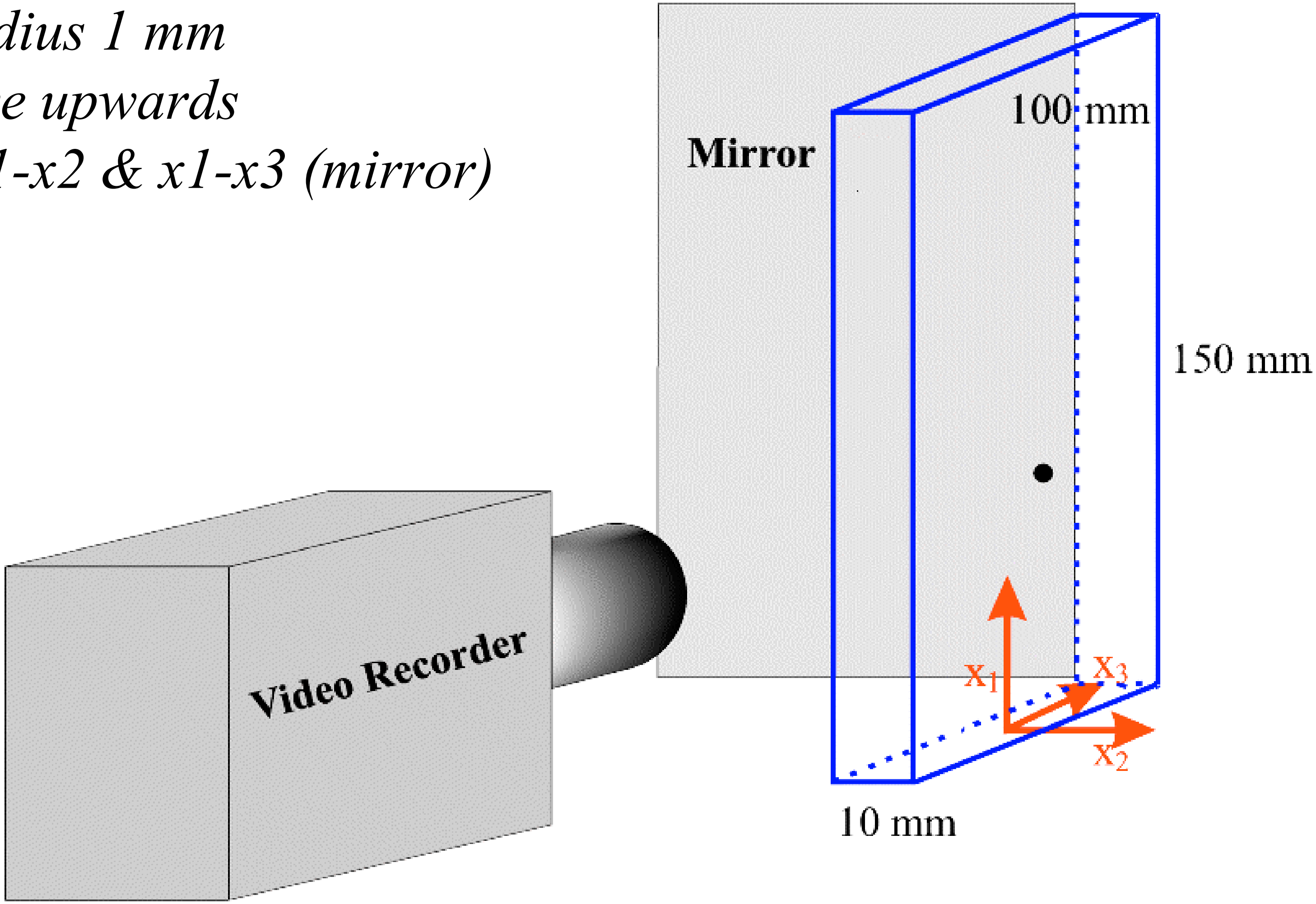


Good Agreement for Micro-Flows
(Low Reynolds number)

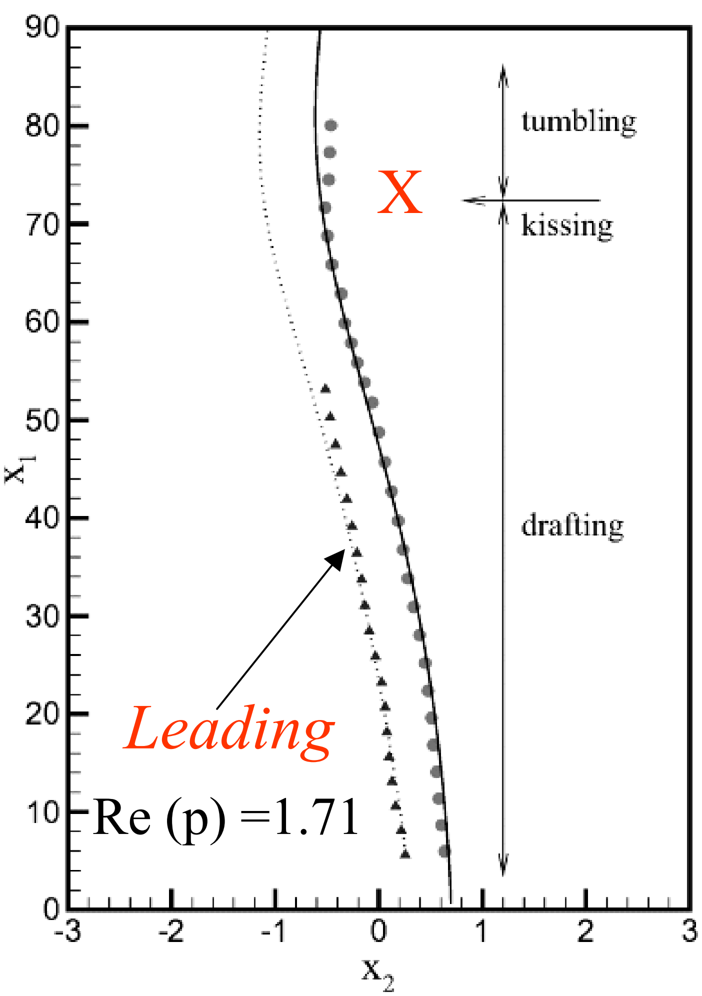
* Dent & Maxey, 1999

Experimental Setup *(Lomholt et al., RISO)*

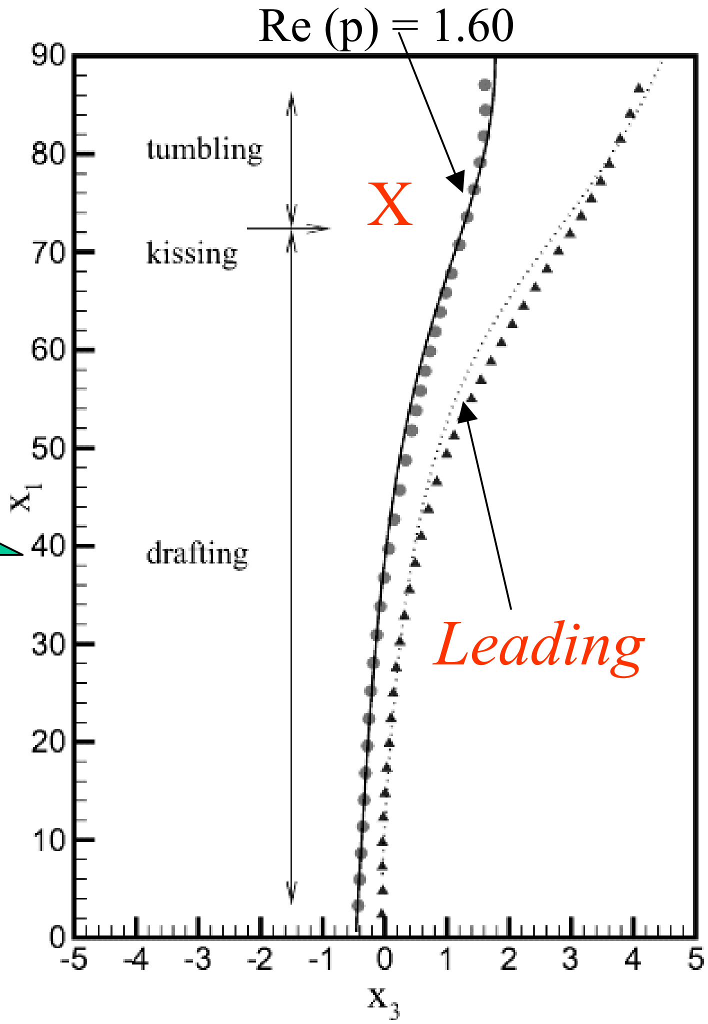
- *Fluid: Glycerol + Water for low Re*
- *Spheres of radius 1 mm*
- *Particles move upwards*
- *Two views: x1-x2 & x1-x3 (mirror)*



Simulation versus Experiment: Two Particles



X1-X2 View

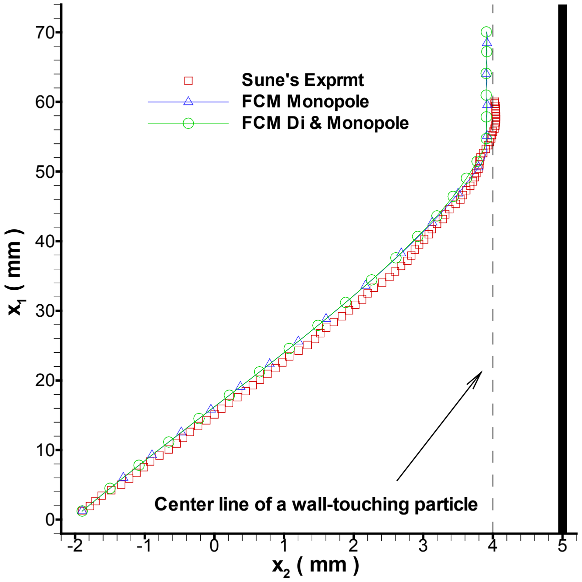


X1-X3 View

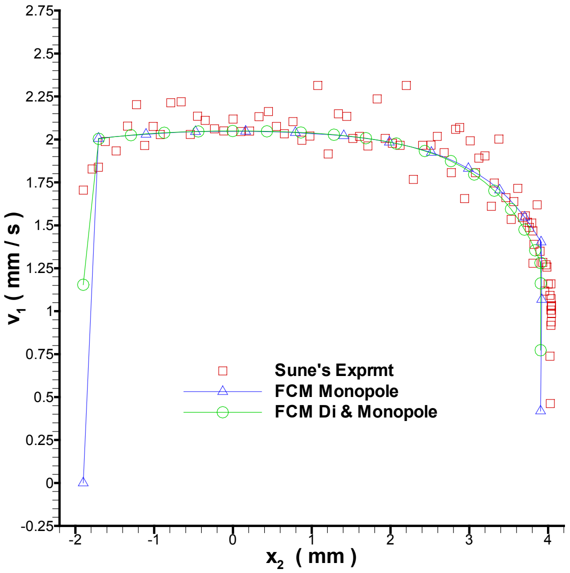
(Lomholt et al, RISO)

Simulation versus Experiment: Re = 0.044

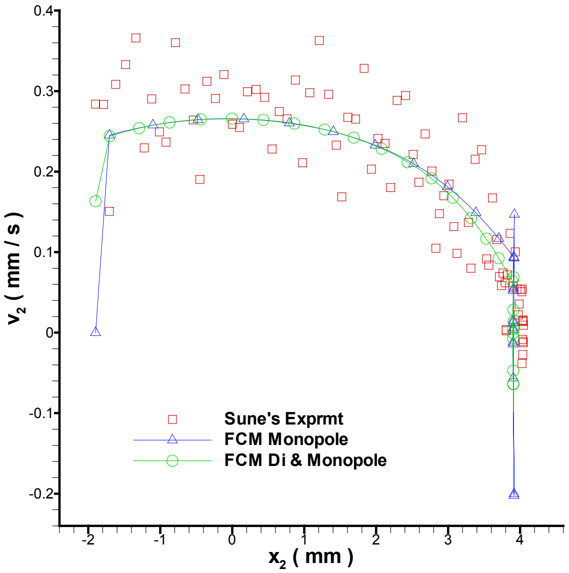
(Sune Lomholt et al, RISO)



- Trajectory

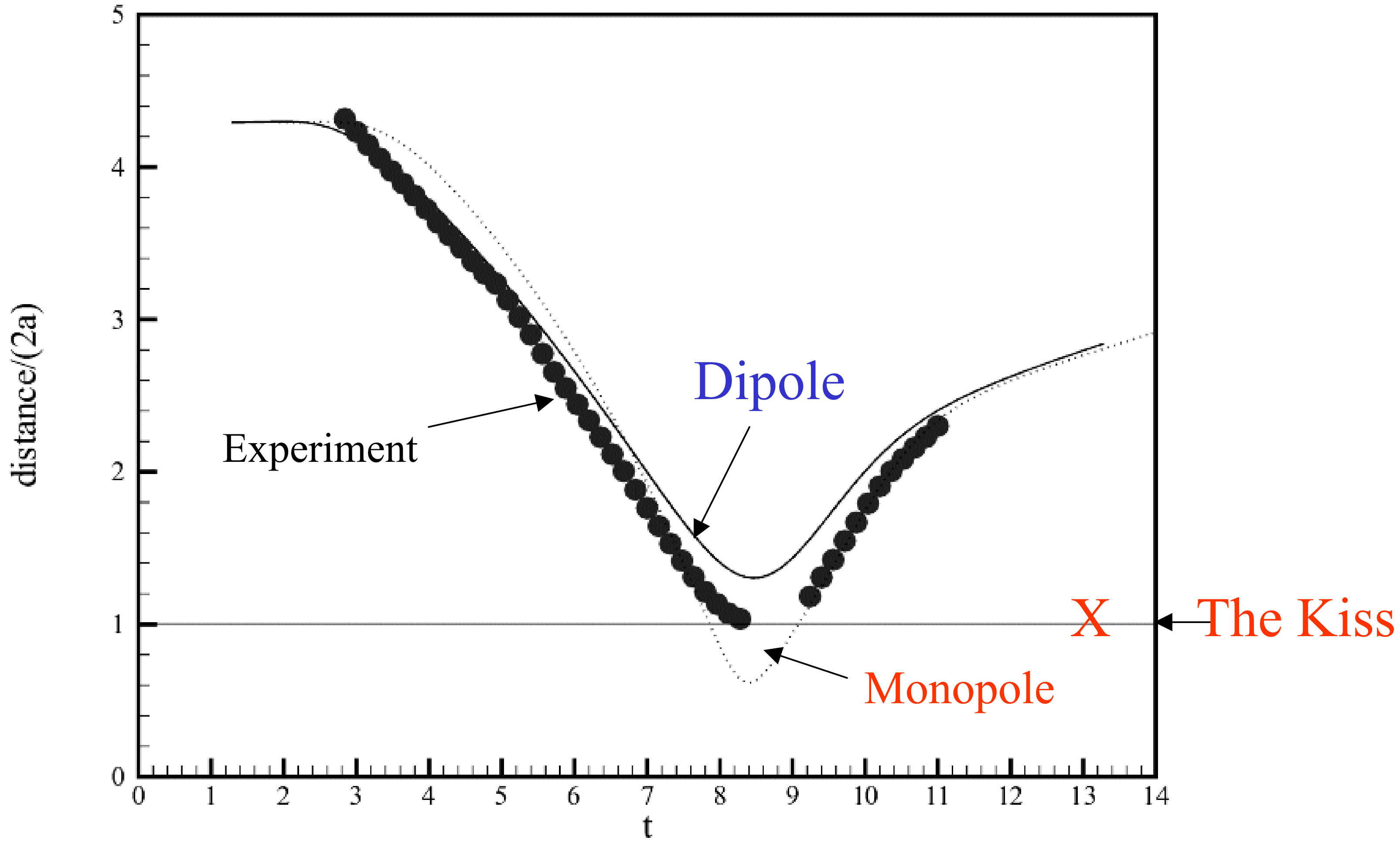


- Parallel to Wall Velocity



- Normal to Wall Velocity

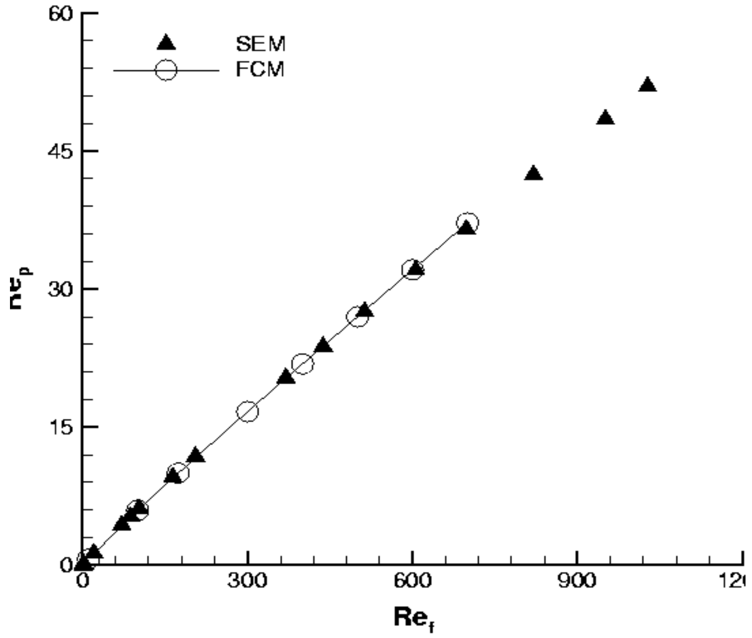
Monopole versus Dipole – Wall Effects



•Distance between two particles as function of time

(Lomholt et al., RISO)

Comparison between SEM and FCM (Particles in a Periodic Lattice)



Good Agreement for Micro-Flows
(Low Reynolds number)

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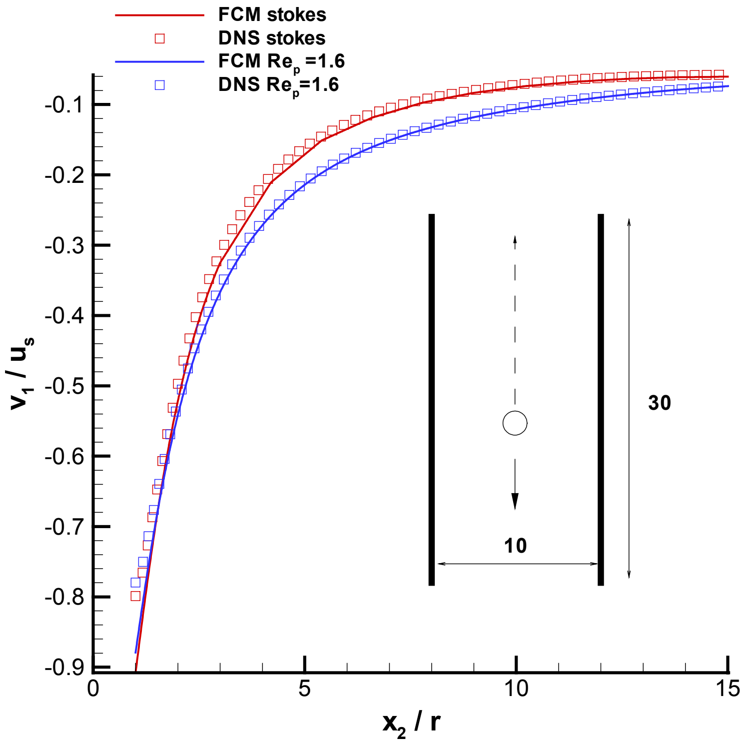
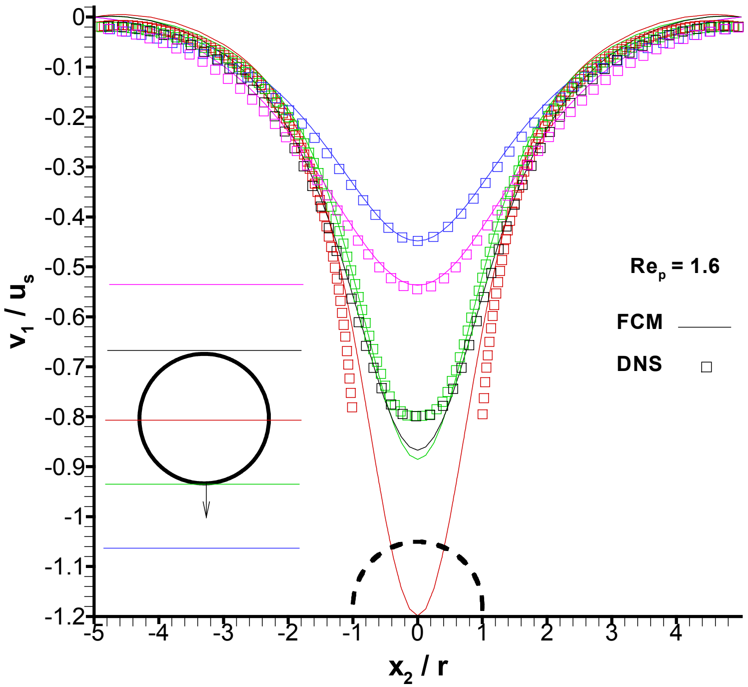
FCM versus Spectral DNS

Particle Re =1.6



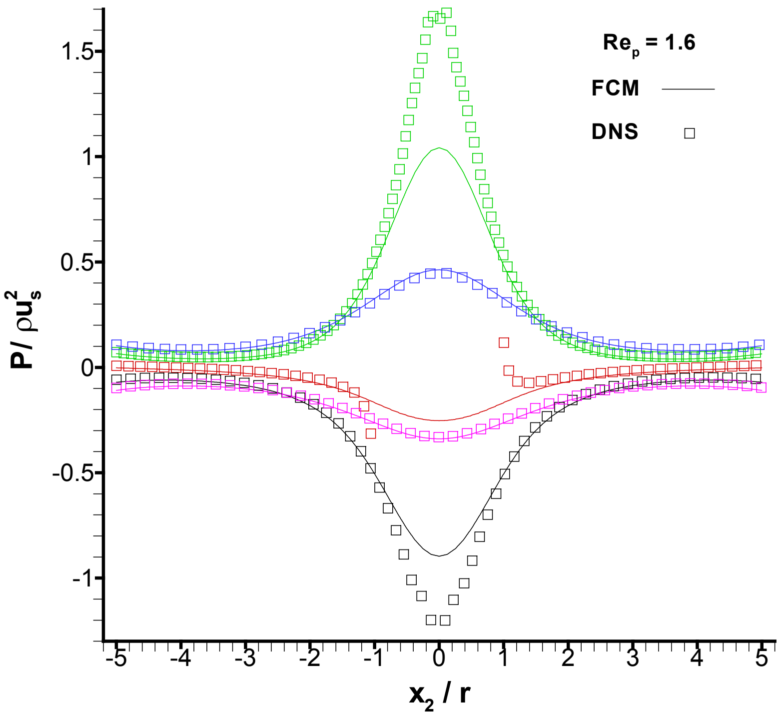


- Velocity Profiles

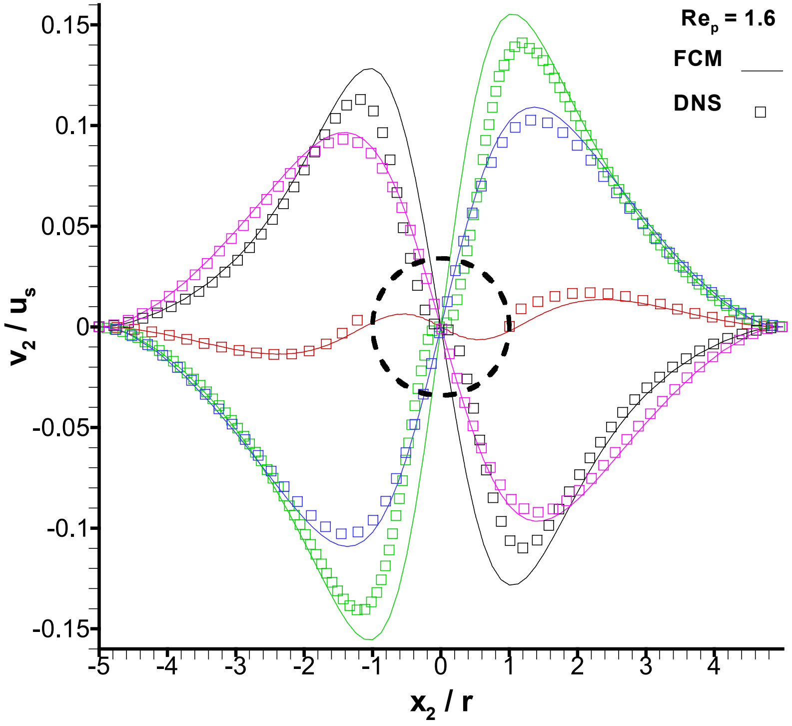
(Stokes versus Navier-Stokes)

- Streamwise Velocity

FCM versus Spectral DNS:
Detailed Velocity Comparisons



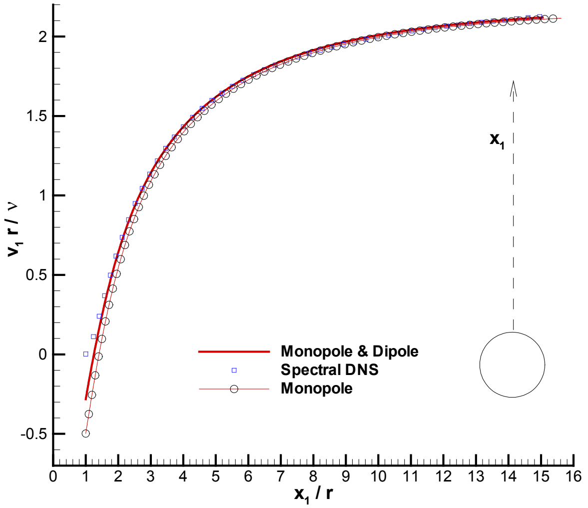
- Pressure at different locations (finite Reynolds number)

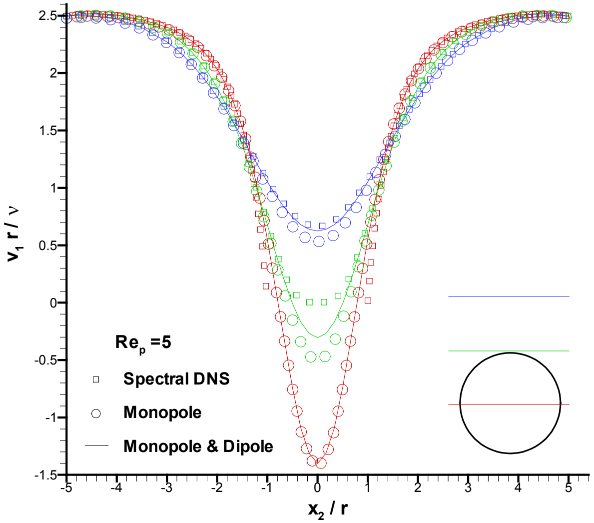


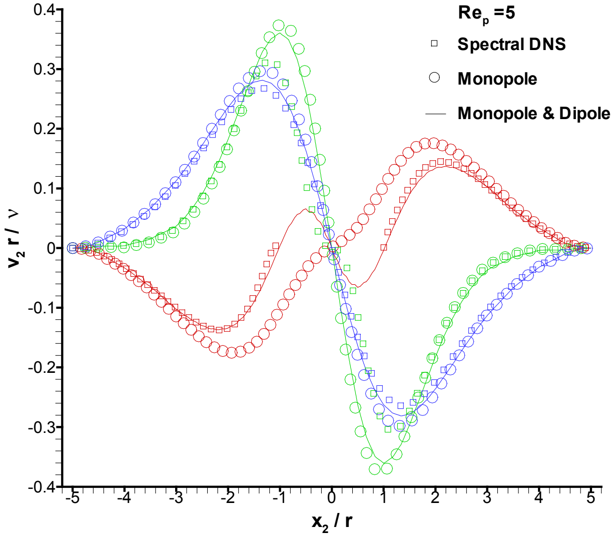
- Crossflow Velocity

FCM versus Spectral DNS

Particle Re = 5: Velocity Comparisons

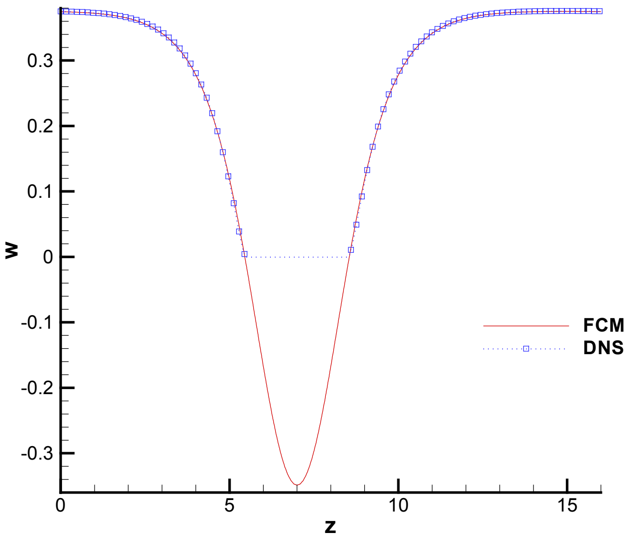


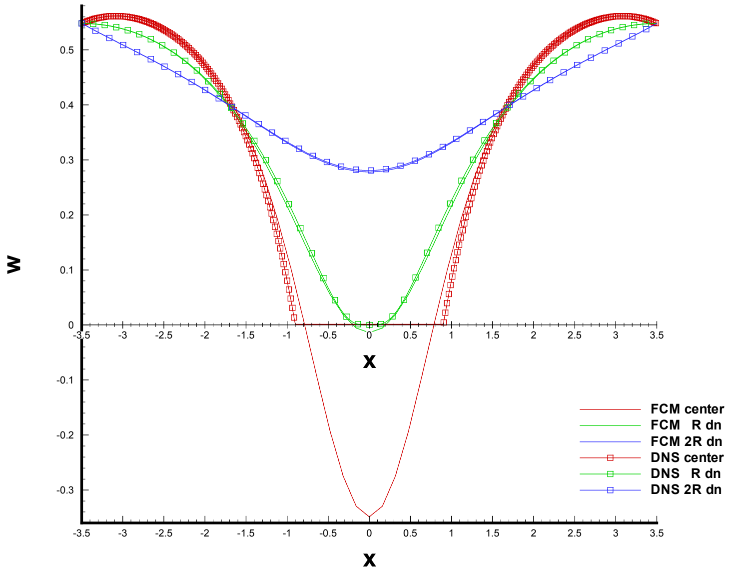


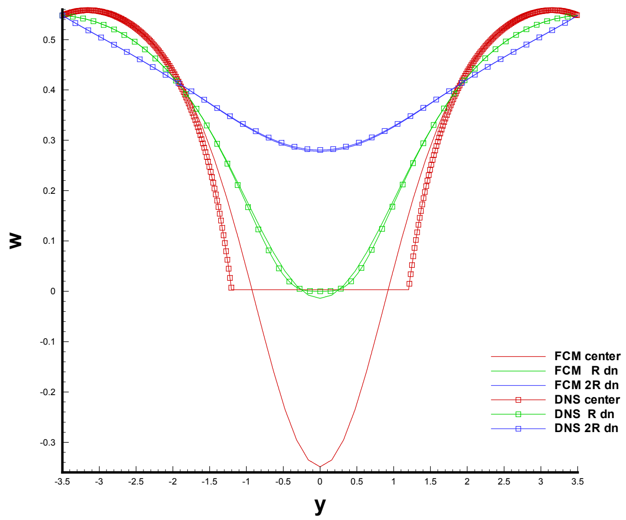


- Velocity far away from particle
- Streamwise Velocity
- Crossflow Velocity

Flow past an ellipsoid aligned in a duct



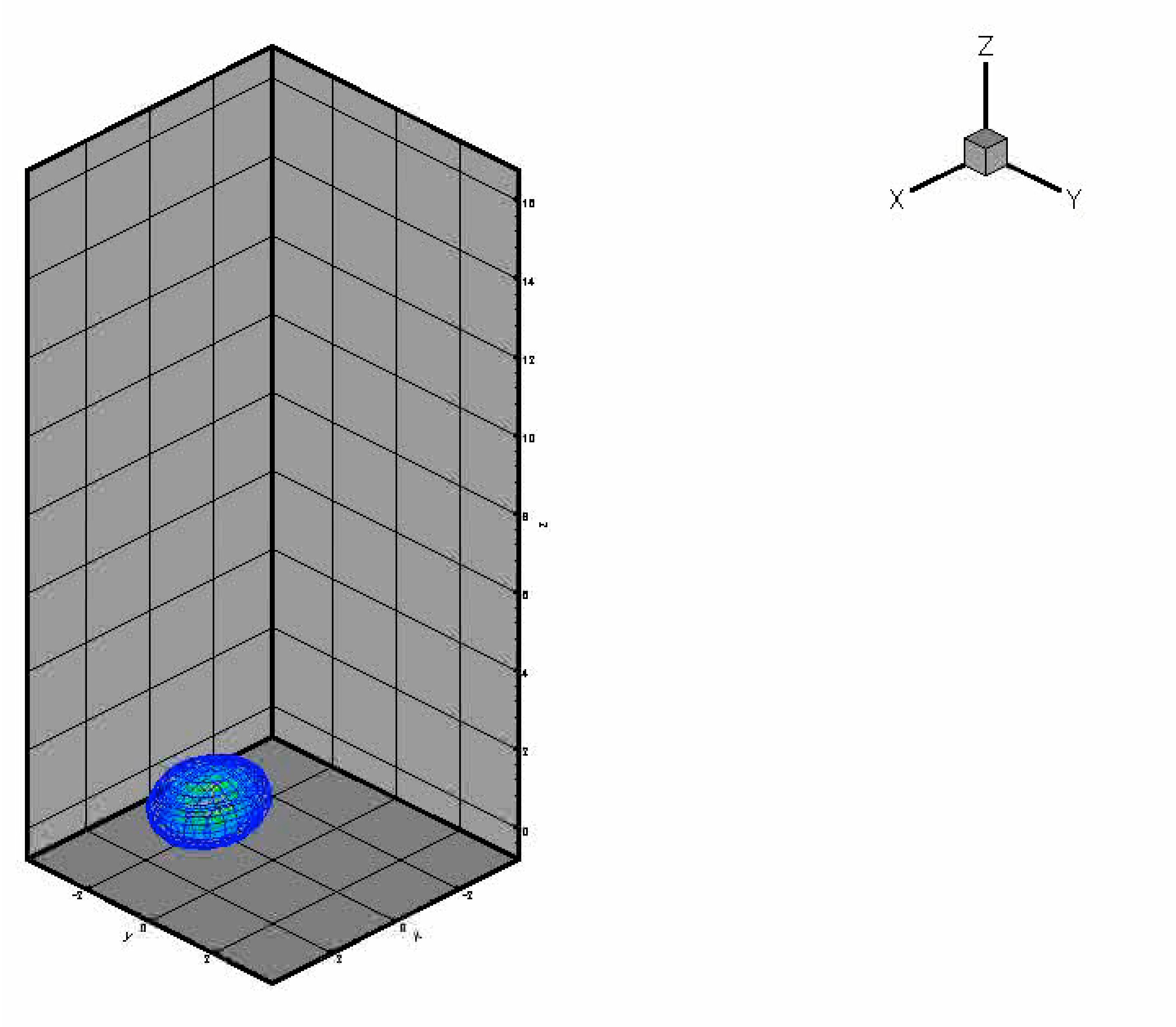




Profiles of axial velocity, w for Re=0:

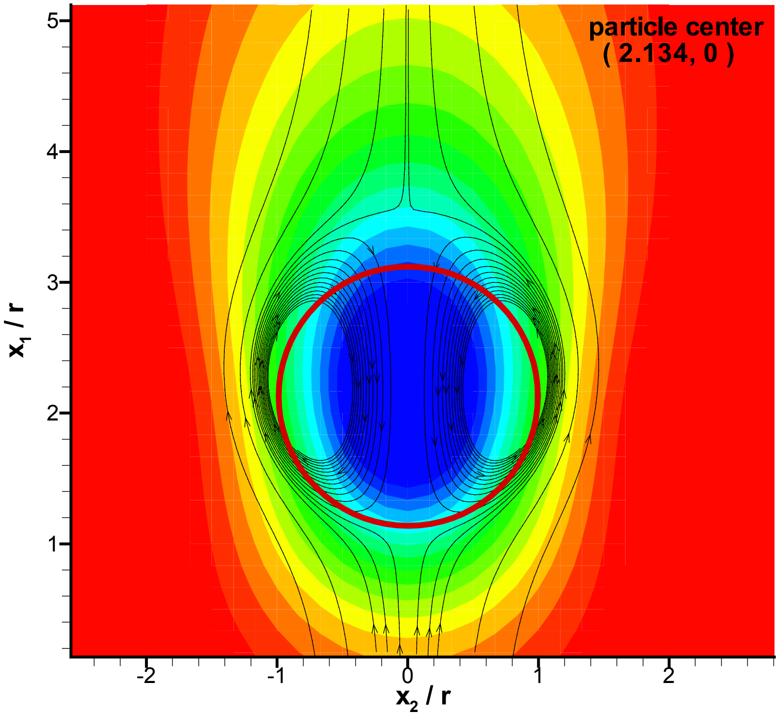
- Along channel z-axis
- Wall normal, x-axis
- Wall normal, y-axis

Ellipsoid in Duct Flow



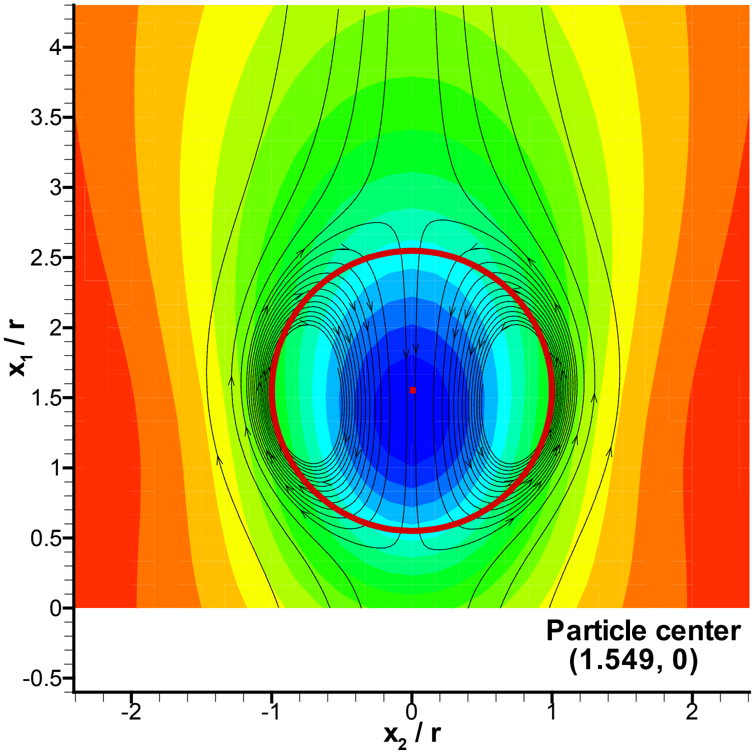
FCM versus Spectral DNS

Particle Re = 5: Flow Field Detail



- Using Monopole Only

Particle is off-centered
relative to the stream lines

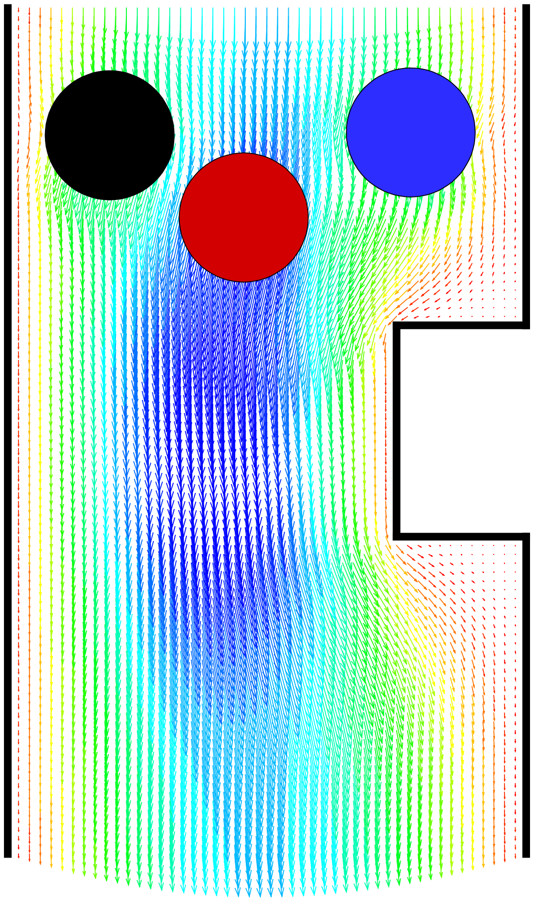


- Using Monopole & Dipole

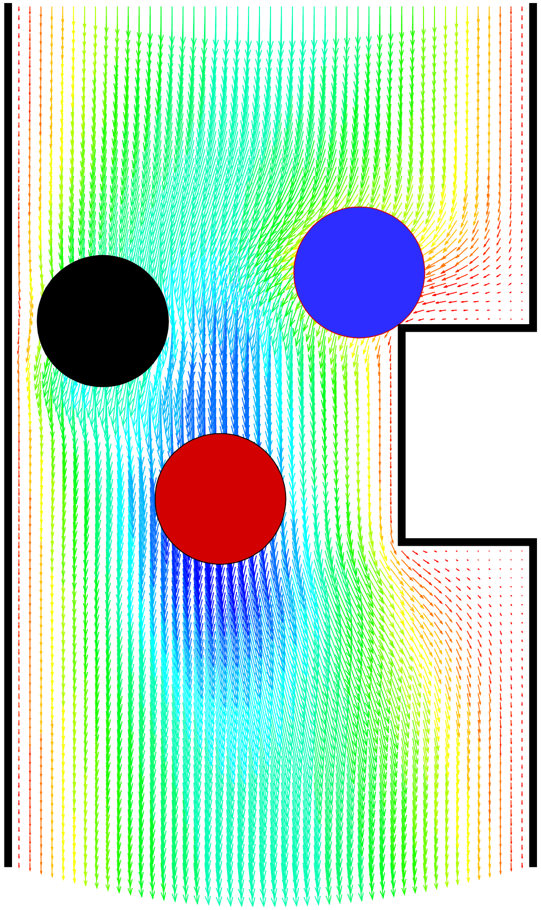
Particle is re-centered
relative to the stream lines

Complex Geometries: Poiseuille Flow

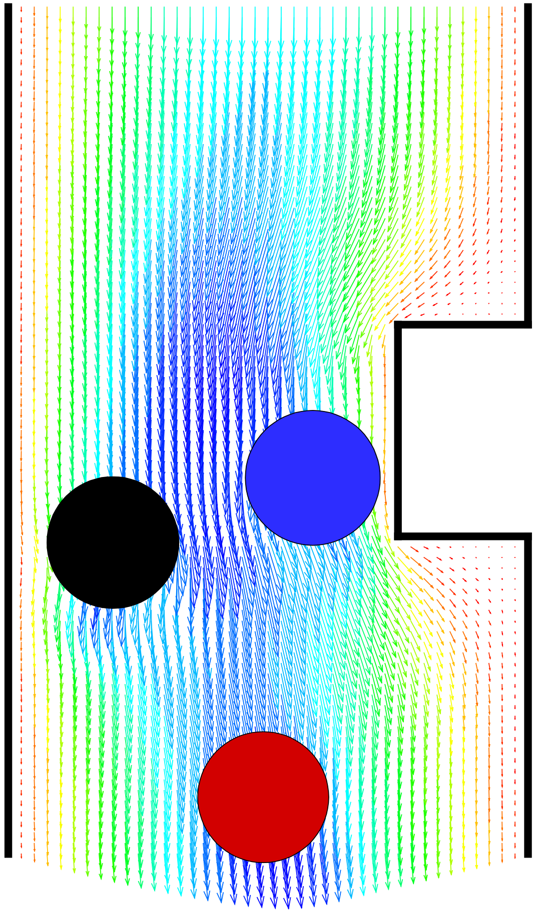
Particle Inertial Included: Density Ratio = 2



- T=2

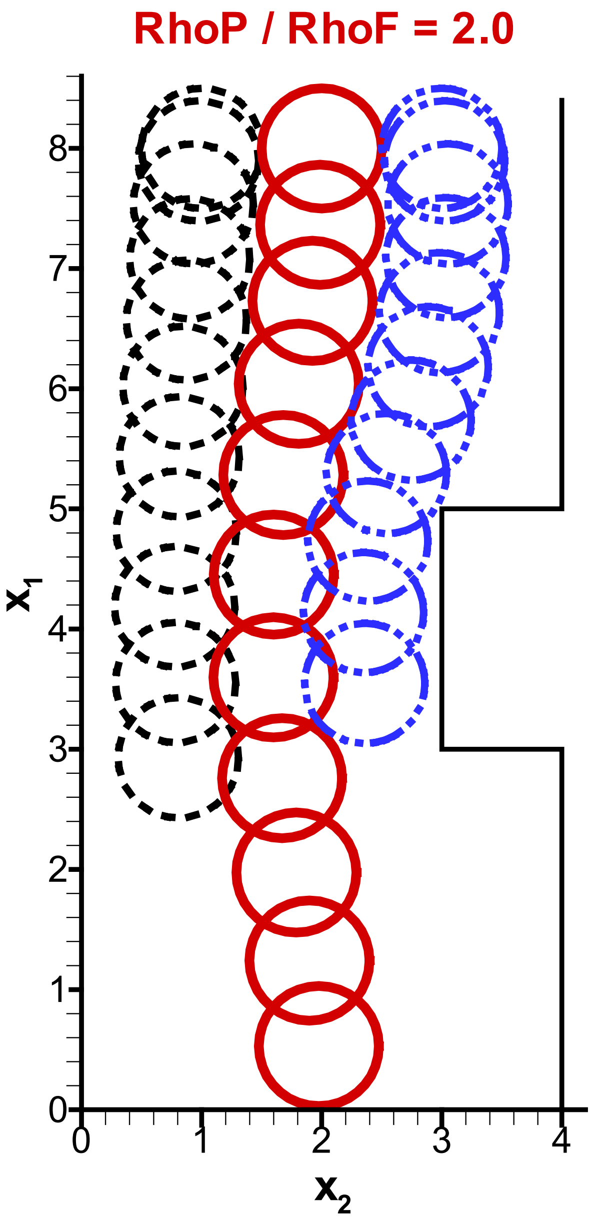


- T=4

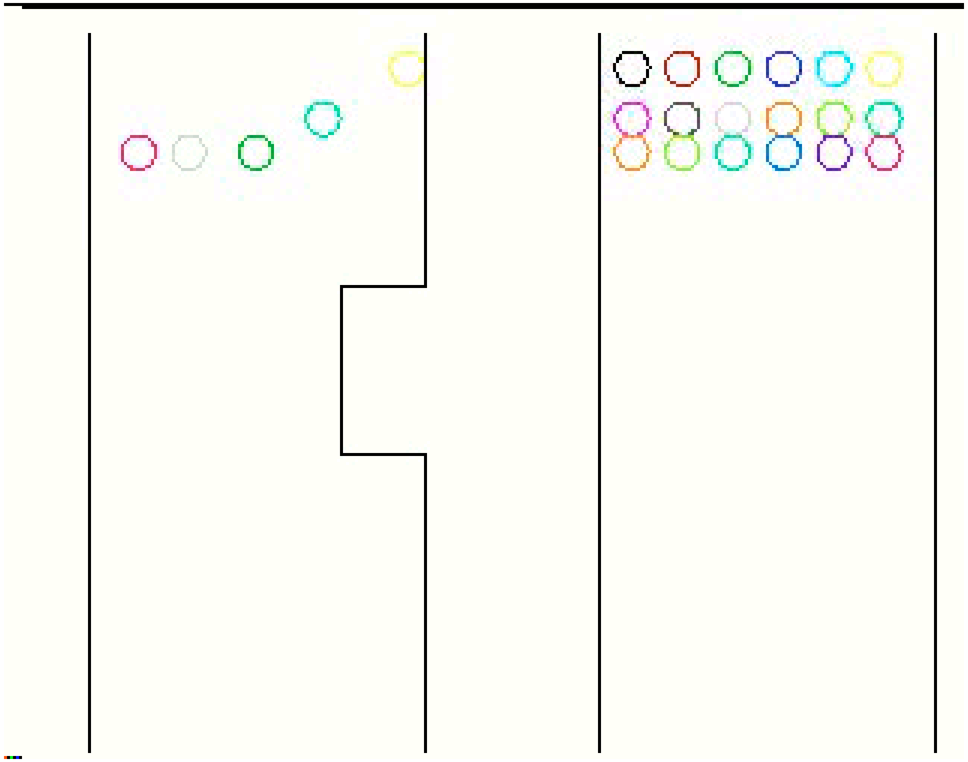


- T=6

Particle Trajectories in Complex Channel



Complex Geometries



Computational Cost

	Number of Elements	Number of Modes	CPU time per step	Number of Processors	Total CPU Time	Machine Type
DNS	**3582**	**9**	**16 sec.**	**64**	**13 hrs**	**IBM**
FCM	**3600**	**4**	**58 sec.**	**1**	**52 mins**	**PC P4**

- CPU Percentage for 2 particles:
  - Main flow ----------------- 95%
  - Monopole ----------------- 2%
  - Dipole & Monopole ----- 5%
- 100 particles or less, 10-15% CPU time overhead
- 500 particles, extra 25% CPU time needed

Efficient Modeling of Contact Forces

Force barrier

- Repulsion force increases as bubbles approach
- Effective for slow to moderate collisions

$$\mathbf{F}^{AB} = ffrac \times \frac{(\mathbf{Y}^A - \mathbf{Y}^B)}{d} \times \left(\frac{(r_0^2 - r^2)}{(r_0^2 - d^2)} \right)^2, r < r_0$$

$$\mathbf{F}^{AB} = 0, r > r_0$$

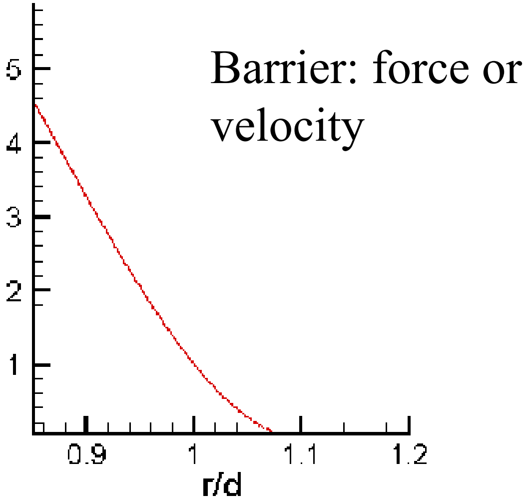
$$r_0 = farfac \times d$$

$$r = \left| \mathbf{Y}^A - \mathbf{Y}^B \right|$$

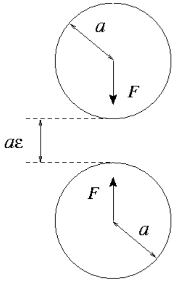
ffrac =force at contact

Velocity barrier

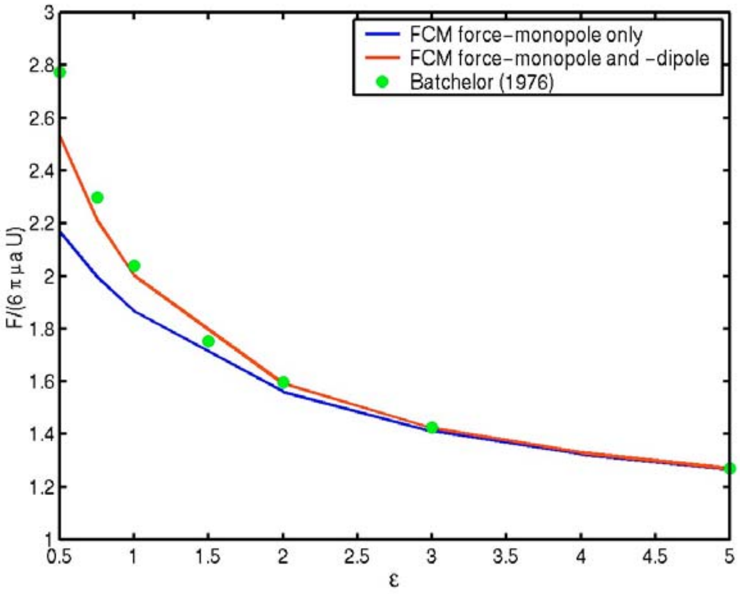
- Repulsion velocity added to each bubble
- Effective for fast to moderate collisions

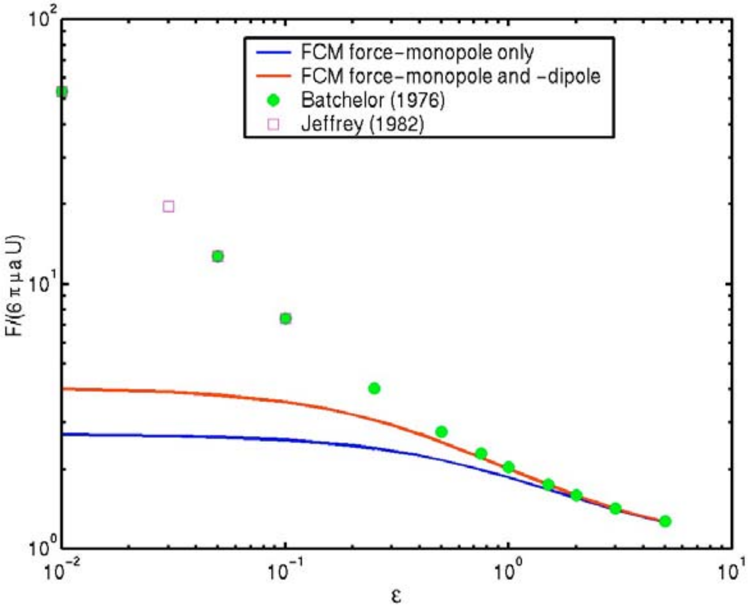


COLLISION MODELS - CASE STUDIES:
Interaction of Two Spheres -- Squeeze



* Dance, PhD (in progress)

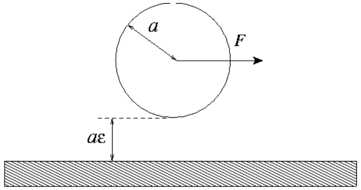




- Force calibration (Linear Scaling)

- Very Small Distances (Log Scaling)

COLLISION MODELS - CASE STUDIES:
Interaction of Sphere with Wall



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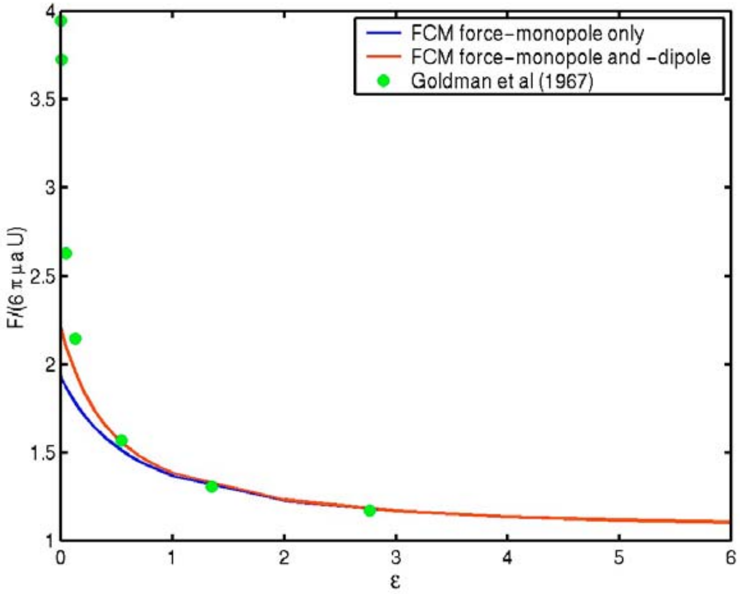

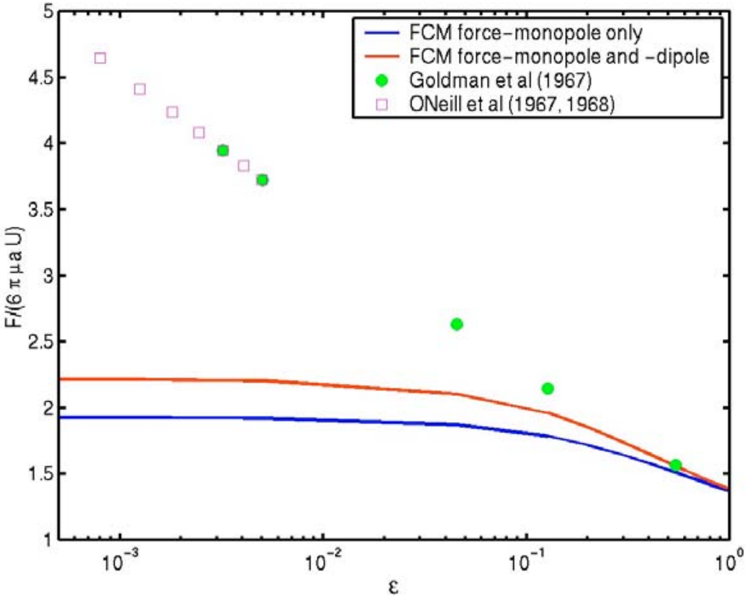

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