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#### SIMONS FOUNDATION Benchmarking high accuracy Set the set of the Collaboration on the many-electron problem Presented by Lucas K. Wagner

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### How reliable are first principles calculations?





#### DFT, DMRG, QMC, GW, etc

#### Total energy, excitations, properties

+  $H|\Psi\rangle = i\hbar\frac{\partial}{\partial t}|\Psi\rangle$ 





How close are these to exact solutions?









### Previous work from the Collaboration



LeBlanc et al.

Agreement between high accuracy methods on the Hubbard model



Motta et al Chain of hydrogen atoms





# Our objective

#### Provide total energy benchmarks for realistic systems on well-defined Hamiltonians.

exhibit strongly correlated behavior in materials

Simplify: ground state energy.

# Important systems: transition metal compounds, which





# Choosing a Hamiltonian

#### Frozen core

- •Complex to specify
- Not necessarily more accurate than ECPs

#### All electron

- Very expensive
- DMC is hard

#### ECPs (Trail and Needs)

- •Everyone can do it
- Accurate and available





### Finite basis sets

$$\hat{H} = -\frac{1}{2}\sum_{i}\nabla_{i}^{2} + \hat{V}$$

Project onto discrete basis

$$H_{ij} = \langle i | \hat{H} | j \rangle$$

Sparse representation

$$\hat{H} = \sum_{ab} t_{ab} c_a^{\dagger} c_b + \sum_{abcd} V_{abcd} c_a^{\dagger} c_b^{\dagger} c_c c_d$$

	For a single Cr atom:				
	Number of one- particle orbitals	Approximate size o Hilbert space			
vdz	54	<b>10</b> <sup>15</sup>			
vtz	88	<b>10</b> <sup>19</sup>			
vqz	135	<b>1</b> 0 <sup>21</sup>			
v5z	197	<b>10</b> <sup>24</sup>			
cbs	infinite	infinite			







### Methods and (roughly) governing equations

**Density functional theory** LDA, PBE, SCAN, HSE06

**Green's function** OSGW, SC-GW, GF2, RPA

Many-body wave function AFOMC, CCSD, CCSD(T), CIC+Q, CISD, DMC, DMRG, FCIQMC, MRLCC, SHCI

Embedding SEET

 $E = f[\rho]$ 

 $G = G_0 + G_0 \Sigma G$ 

 $\hat{H}|\Psi\rangle = E|\Psi\rangle$ 





#### **PySCF (checked Molpro)**

Density functional theory Coupled cluster **Configuration** Interaction DMRG Integral generation ¬

#### **QWalk (checked CHAMP)**

Fixed node diffusion QMC

#### MolGW (Bruneval)

QSGW (van Schilfgaard)

#### Home-grown codes

Self-consistent GW (Gull)

- Semistochastic heat bath iteration (Umrigar)
- Auxiliary field QMC (Zhang)
- Multi-reference linearized coupled cluster (Sharma)
- Full CI OMC (Booth)







### Database

#### Sc, Ti, V, Cr, Mn, Fe, Cu, 0,+,M-O

# vdz, vtz, × vqz, v5z $\mathbf{X}$

18 TM systems neutral atoms, ions and oxides

5 basis sets



#### 20 methods









Total ener

### Analysis of data (Cr atom)

	$\bigcirc$	vdz
	$\bigcirc$	vtz
	$\bigcirc$	vqz
	0	v5z
	0	$\operatorname{cbs}$
prrelation energy in a basis		
orrelation energy		
-86.5 $-86.4$ $-86.3$ $-86.2$		
rgy (Hartree) SIMONS F	ΟU	NDA





### Setting a reference

FCIQMC

DMRG

SH





RMS deviations between 3 methods are much less than 1 mHa, around chemical accuracy.



### How methods agree: cluster analysis



## Basis set extrapolation: total energy



Basis set error scales as ~  $1/z^3$ 

Diffusion Monte Carlo uses the full Hamiltonian, but is approximate; separate lower bound on correlation energy.

Approximate uncertainty in extrapolated value of 2-5 mHa.

DFT results are compared only in the CBS limit.





### Assessing performance: correlation energy



Most methods obtain consistent percentages of the correlation energy.

Scale-free quantity that allows us to compare different systems with different total energies





# Correlation energy performance





Energy differences

## Basis set extrapolation: differences

#### Ionization potential of Cr









#### Energy differences: the difficulty with experiment



Experimental uncertainties are substantial

Measure the distance from the range of experimental results









### Limiting factor is experiment!







### Deviation from experiment



Deviation from SHCI reference (eV)

		Ionizatio	n potenti	al				
		Dissociat	ion energ	gy –	_			
				_				
				_				
				_				_
				_				
				-		0		
						8		
				_				
						÷		
					0			
						0		
				_				
				_				
						0		
	I		I					
	4		6		-	-0.25	0.00	0.25
$(\mathbf{o}\mathbf{V})$								





# Summary

vdz, vtz, vqz, v5z Sc, Ti, V, Cr, Mn, Fe, Cu

- Provenance well-defined
- •Internal reference, precise results on systems containing up to 25 correlated electrons



- •Enables clean method assessment
- Practical calculations rely on cancellation of error to obtain highly accurate results



















#### DMC(SD)













## Total energy in a basis



Error is **only** the error in the solution method.

Useful for debugging and improving methods





# Correlation energy

(a) Atoms



Many methods obtain very similar percentage of correlation energy across different systems and basis search and basis search and basis search and basis search are searched as the search and basis searched by the searched basis of the searched basis are searched by the searched basis of the searched basis are searched by the searched basis of the searched basis are searched by the searched basis of the searched basis are searched by the searched basis of the searched basis are searched by the searched basis of the searched basis are searched by the searched basis of the searched basis are searched by the searched basis of the searched basis are searched by the searched basis of the searched basis are searched by the searched basis of the searched basis are searched by the searched basis of the searched basis are searched by the searched basis of the searched basis are searched by the searched basis of the searched basis are searched by the searched basis of the searched basis are searched by the searched basis are searched basis are searched by the searched basis are searched basis are searched by the searched basis are searched by the searched basis are searched basis are searched by the searched basis are searched basis are searched basis are searched bas

#### (b) TM-O molecules





## Ionization relative to experiment



vdz vtzcbs34cbs345cbs45cbsExperiment  $\leq 10~{\rm mHa}$  $\leq 3 \text{ mHa}$ 

Experimental values very accurate.

Reference methods in extrapolated basis achieve "chemical accuracy" of 1 mHa.

Effective core potentials, spin-orbit effects lead to SSMAL ARGORSOUNDATION







# Disocciation energy











## Did we test "strong correlation"

The systems we tested were not what chemists would call "strongly correlated."

The 1-RDM for an accurate result is not too different from the 1-RDM of a single determinant wave function.

However, so-called "dynamic correlation" is very large in these systems.

This is why methods like CCSD(T) work very well but multi-reference methods perform worse; they have "wasted" some of their description space.





### Other thoughts

One can implement correction methods based on the consistent performance of some methods (FN-DMC, for example)



