Synchronization and Extreme Fluctuations on Networks, and Application to Scalable Parallel Discrete-Event Simulations

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Synchronization in Coupled Multi-Component Systems

Collective dynamics on the network

Examples:

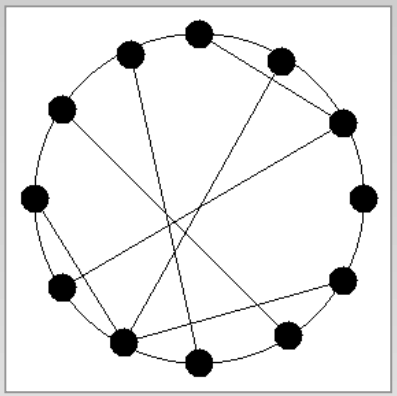
- Internet (packet traffic/flux)
- Load-balancing schemes (job allocation among processors)
- Electric power grid (power transmission and phase synchronization)
- High-performance or grid-computing networks (progress of the processors in distributed simulations)

Fluctuations of the "load" in the network

- Average size of the fluctuations
- Typical size of the largest fluctuations;
  failures/delays are often triggered by extreme events

Critical phenomena in small-world networks

- Watts & Strogatz (1998):
"… enhanced signal-propagation speed, computational power, and synchronizability".



- Monasson (1999): diffusion on SW networks
- Barrat&Weight (2000), Gitterman (2000), Kim et al. (2001), Herrero (2002), Jeong et al. (2003), Novotny and Wheeler (2004): Ising model on SW networks
- Hong et al. (2002): XY-model and Kuramoto oscillators on SW networks.
- these systems typically exhibit (strict or anomalous) mean-field-like phase transitions
- Hastings (2003): general criterion for mean-field scaling

Synchronization in Parallel Discrete-Event Simulations (PDES)

Parallelization for asynchronous dynamics of continuum-time processes

Examples:

- Cellular communication networks (call arrivals)
- Magnetization dynamics in condensed matter
  (Ising model: spin flips with Glauber dynamic)
- Spatial epidemic models, contact process (infections)

Paradoxical task:

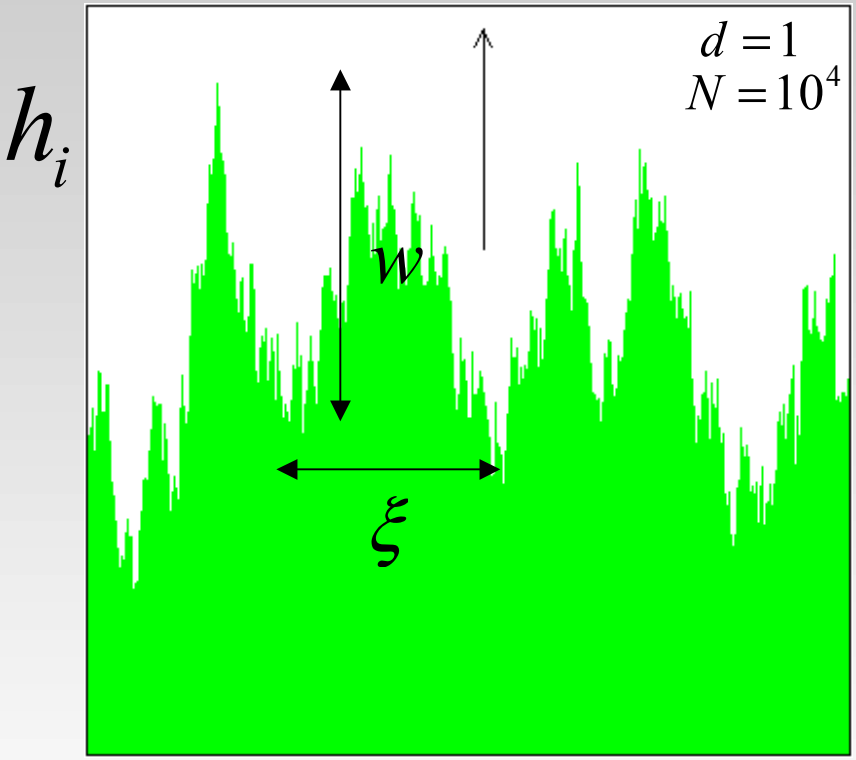
- (algorithmically) parallelize (physically) non-parallel dynamics

Must use local simulated times $\{h_i\}$ on each processor and a synchronization scheme to preserved causality

Lubachevsky (1987)

Synchronization landscape (virtual time horizon)

progress of the simulation



i [index of processing element (PE)]

$$w^2(t) = \frac{1}{N}\sum_{i=1}^{N}[h_i(t) - \bar{h}(t)]^2$$

width (measure of de-synchronization)

"rough" landscape

$$\xi \sim N$$

correlation length

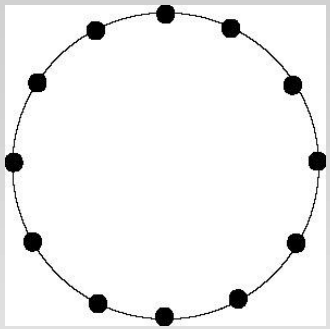
$$w \sim N^{\alpha}$$

$\alpha = 1/2$

roughness exponent

G. K., Z. Toroczkai, M.A. Novotny, and P.A. Rikvold, *PRL* (2000)

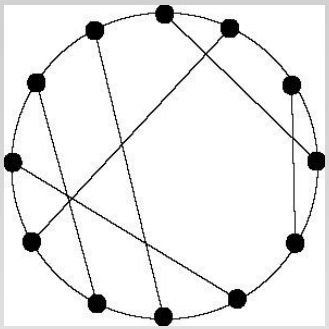
Small-World Synchronized Computing Networks

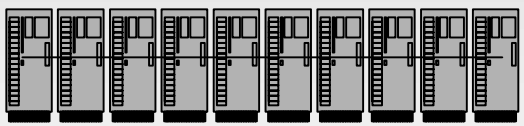


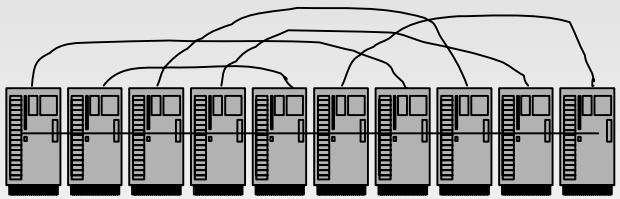
regular network

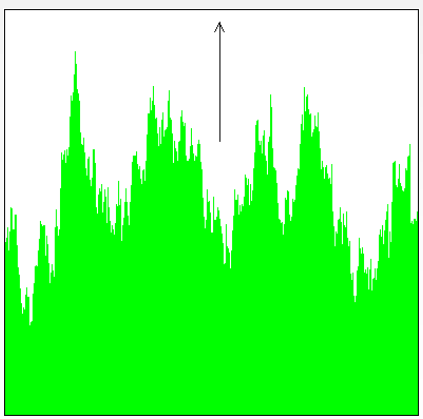
SW network

[o(1) links per node]



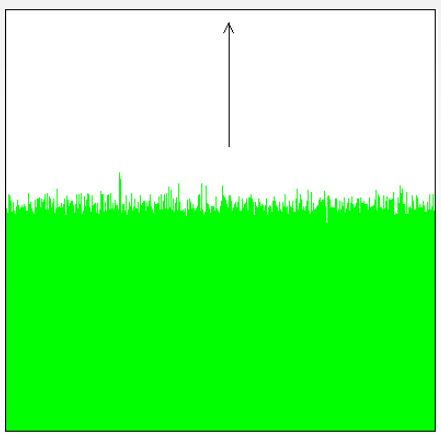






progress of the individual processors

- efficient exchange of information
- no global intervention
- provides scalable scheme for massively parallel computing



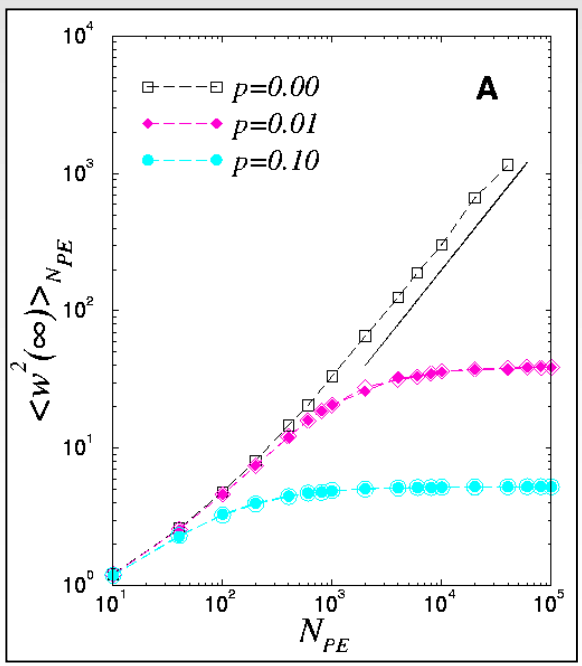
utilization trade-off/scalable data management

⇩

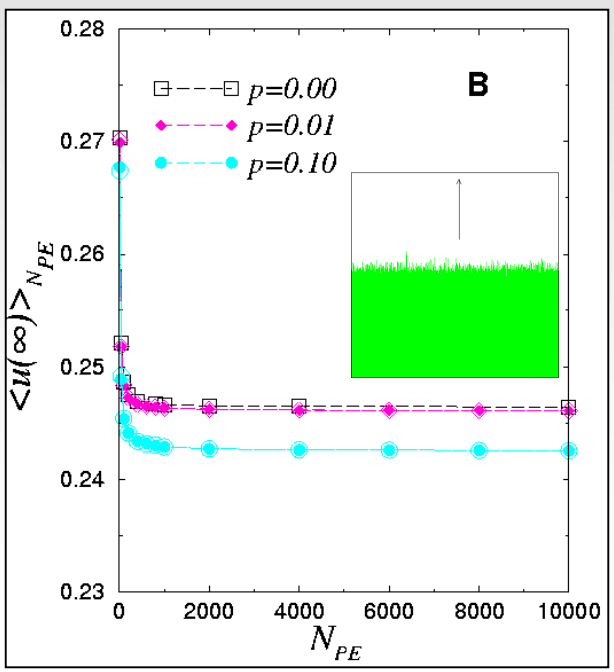
fully scalable high-performance or grid-computing networks

G. K., Novotny, Guclu, Toroczkai, Rikvold, *Science* (2003)
Kirkpatrick, *Science* (2003)

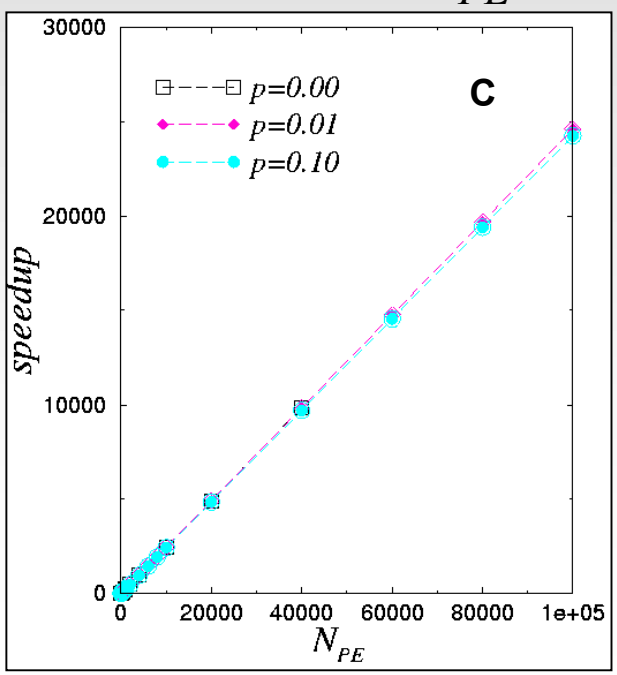
roughness



utilization = fraction of non-idling PEs



speedup = utilization × N_{PE}



local synchronization/communication rules in PDES
("microscopic dynamics")

effective equation of motion for the virtual time horizon

$$\partial_t h_i = (h_{i+1} + h_{i-1} - 2h_i) - \sum_{j=1}^{N} J_{ij}(h_i - h_j) + \ldots + \eta_i(t)$$

Gaussian noise

$$\sum_j J_{ij} = o(1)$$ **finite average degree**

Impurity-averaged perturbation theory

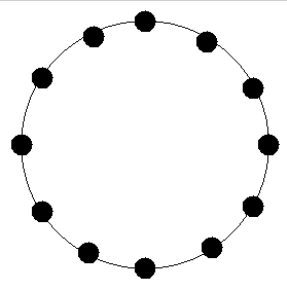
B. Kozma, M.B. Hastings, and G.K, *PRL* (2004)

$$\partial_t h_i = (h_{i+1} + h_{i-1} - 2h_i) - \Sigma(h_i - \bar{h}) + \eta_i(t)$$

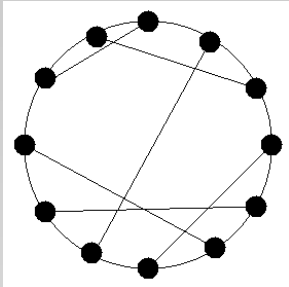
SW network induces **effective relaxation to the mean**

finite correlation length ξ and **finite width** w in the $N \to \infty$ limit

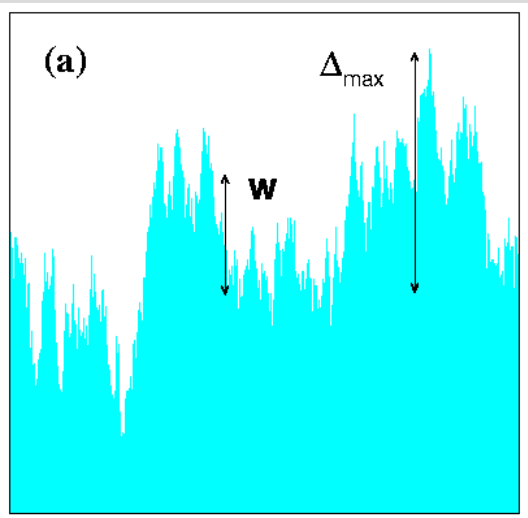
Synchronizability and **extreme fluctuations** in coupled multi-component systems with "local" relaxation and short-tailed noise (e.g., PDES synchronization)



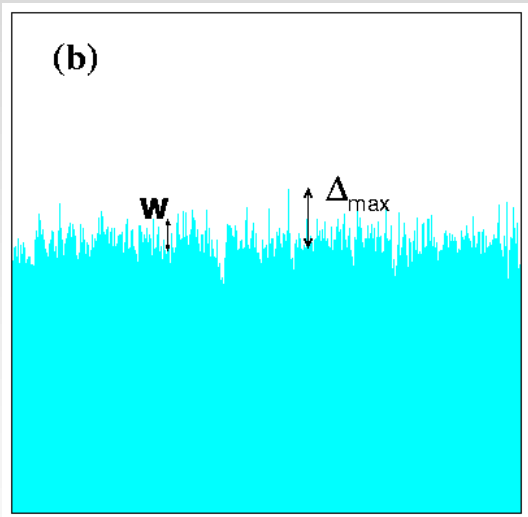
regular 1*d* (ring)



small world



$$\langle \Delta_{\max} \rangle = \langle h_{\max} - \bar{h} \rangle$$



$$w \sim N^{\alpha}$$

$$\langle \Delta_{\max} \rangle \sim N^{\alpha}$$

Raychaudhuri et al., *PRL* (2001)
G.K. et al. *ACM SAC02* (2002)

$N \to \infty$

$$w \sim const.$$

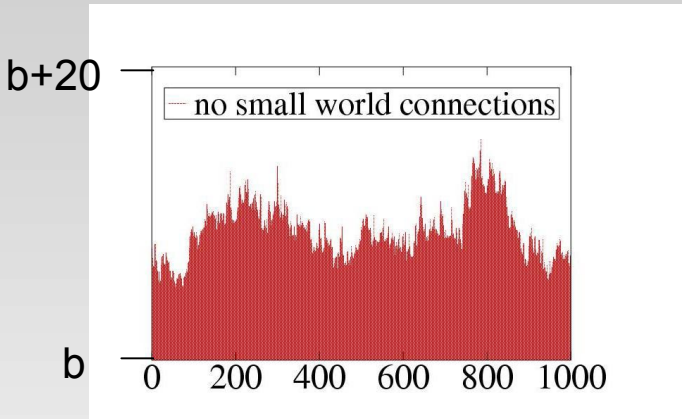
$$\langle \Delta_{\max} \rangle \sim w \ln(N)$$

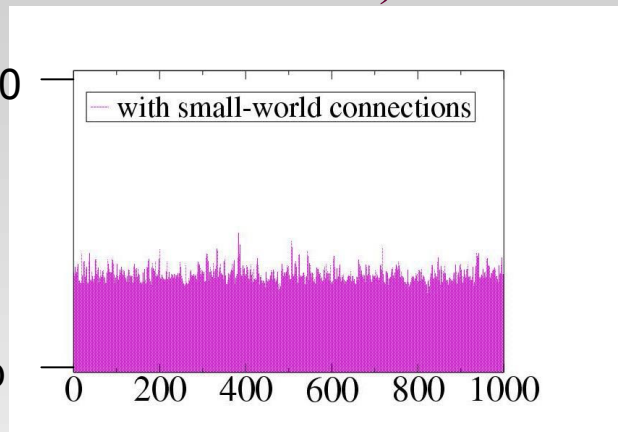
H. Guclu, and G.K., *PRE(R)* (2004)

Dubreus, Novotny, Kolakowska (2004)

Random Walker on (Small – World) Evolving Surfaces

- Random walker moves uphill on nonequilibrium surface
- PDES model (with and without Small-World bonds)

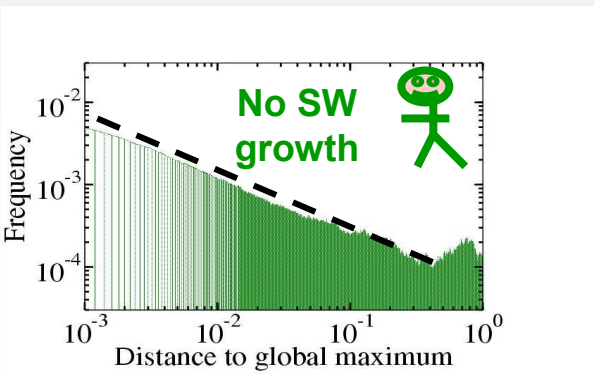




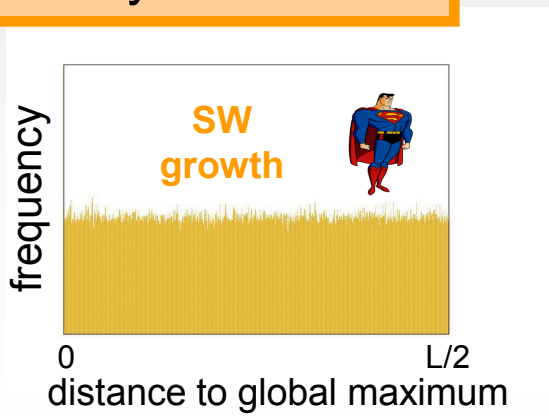
Walker relation to global maximum

Results:

SW connections change the probability distribution



$L = 10^4$



No SW connections: probability distribution is power law

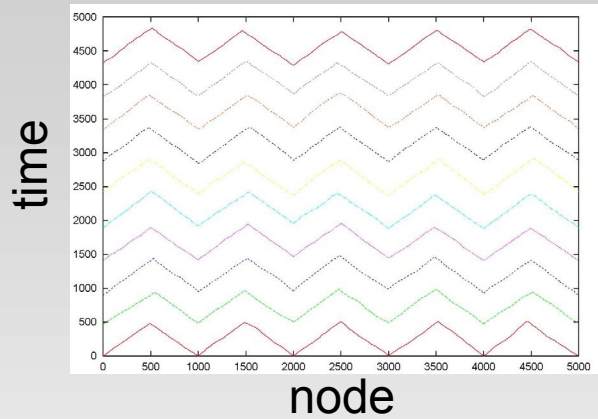
See Poster

Freeze-and-Shift Algorithm

Decouple communications and calculations

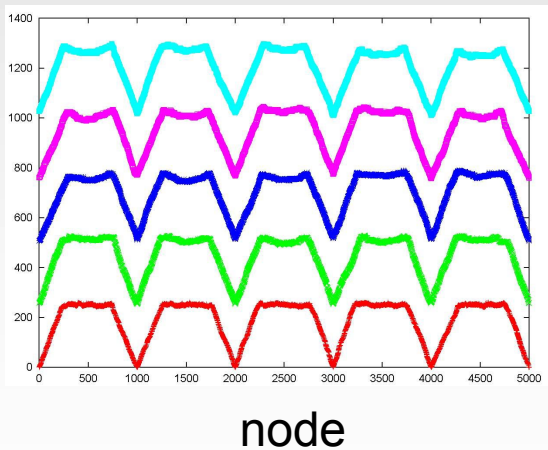
- use treads on each node
- each PE has many nodes & threads
- Freeze boundaries between PEs
- Shift data between PEs

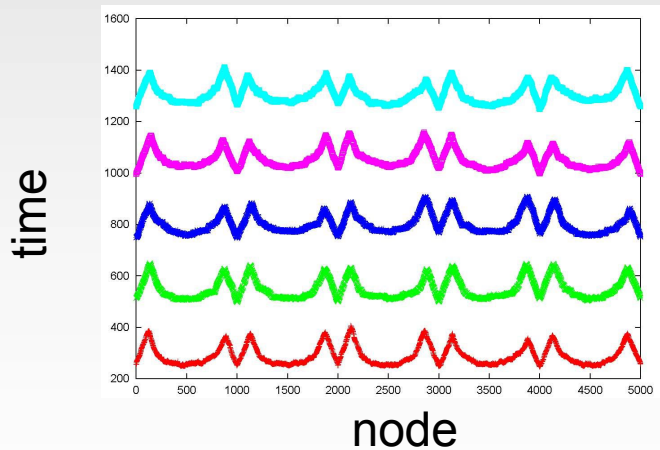
N_{PE}=5 nodes/PE=1000



Shift at full pyramid

Shift at ½ pyramid (2 times)





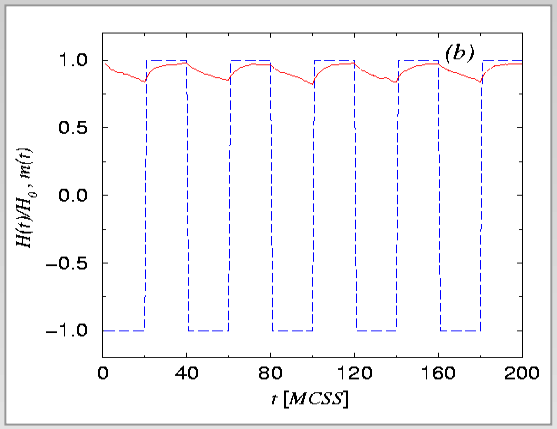
- efficiency near 1
- use for PDES (PDEs? grid computing?)

Shchur and Novotny, *PRE* (2004)

Periodically-driven spatially-extended bistable systems

τ : **metastable lifetime**

$t_{1/2}$: **half-period of the oscillating field**

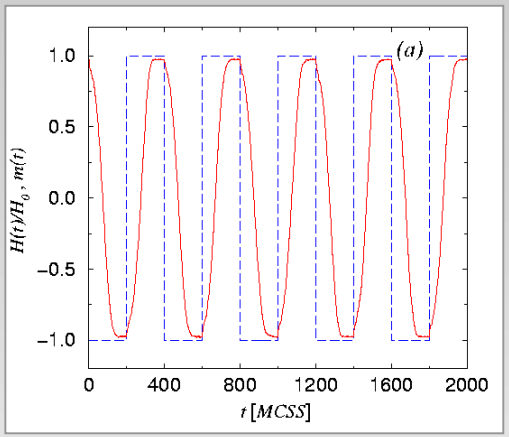


$t_{1/2} < \tau$

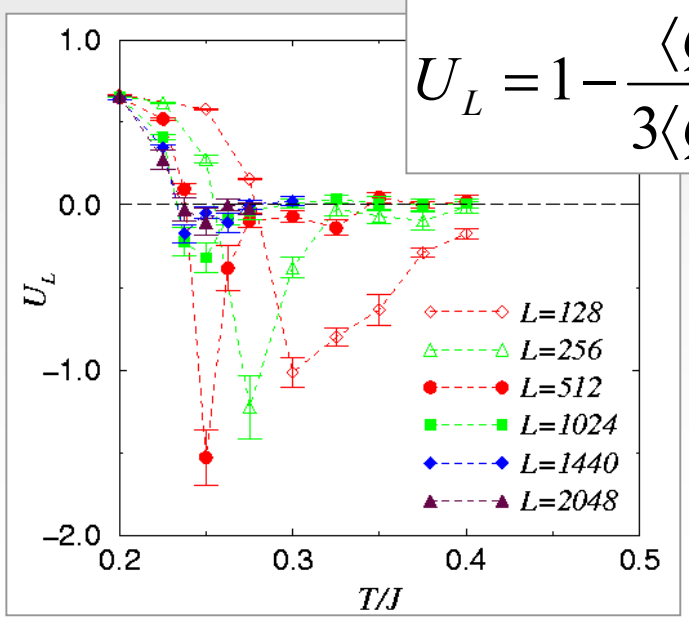
$$Q = \frac{1}{2t_{1/2}} \oint m(t)\,dt$$

dynamic phase transition

$t_{1/2} \approx \tau$



$t_{1/2} > \tau$

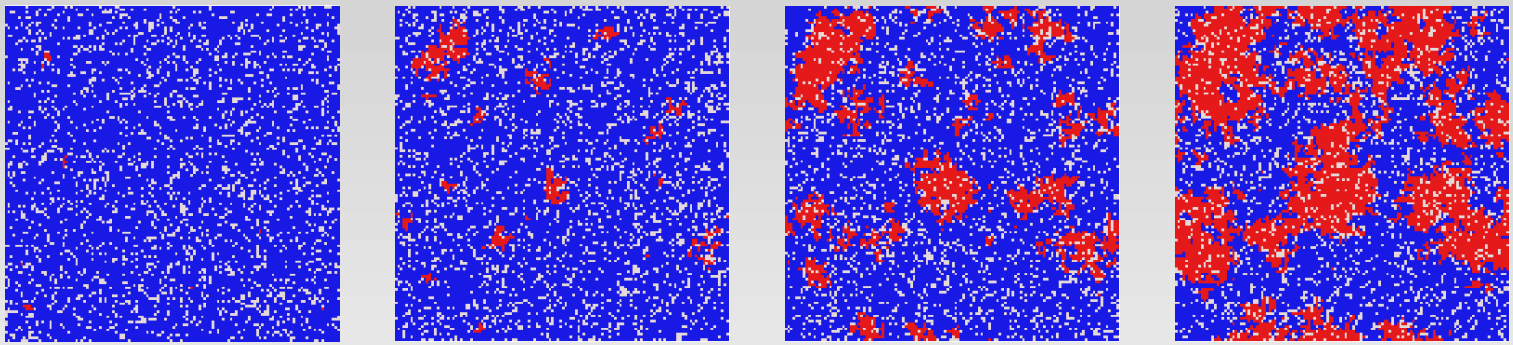


$$U_L = 1 - \frac{\langle Q^4 \rangle}{3\langle Q^2 \rangle^2}$$

- Strong finite-size effects
- No first-order dyn. phase trans.
- No tri-critical point
- SR does not survive as L→∞
- Estimates for cross-overs

G. K., P.A. Rikvold, and M.A. Novotny, *PRE* (2002)

Ecological Invasion with Pre-emptive Competition



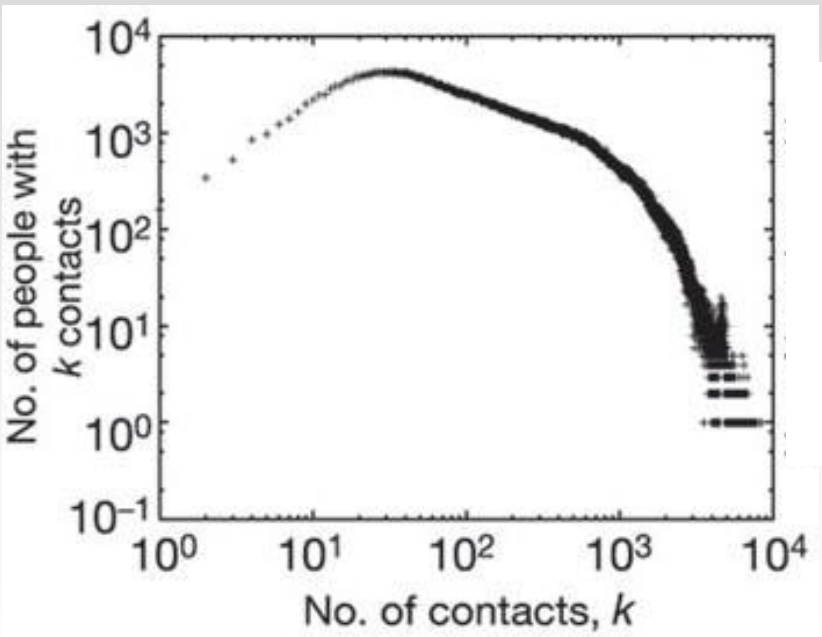
- two plant species compete for shared and limited resources
- initially only **resident species** is present
- **invader species** (better competitor) is introduced stochastically with a small rate

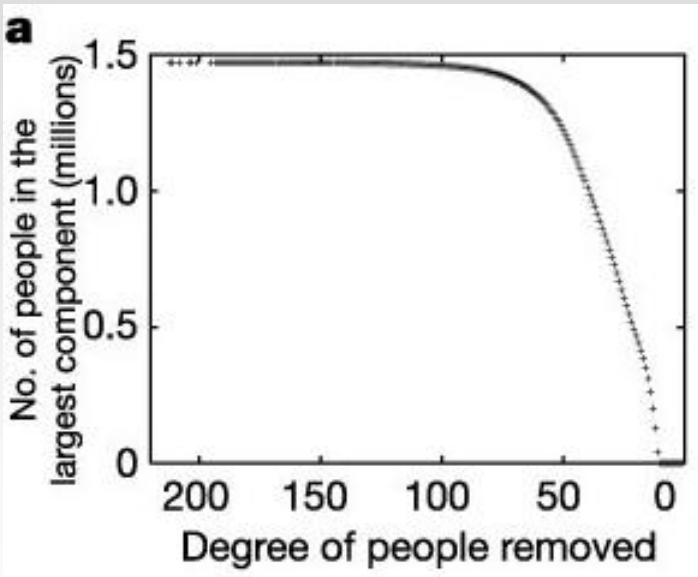
nucleation and growth → **Avrami's Law**

G.K. and T. Caraco (2004)

Modeling Disease Outbreaks in Realistic Urban Social Networks

S. Eubank, **H. Guclu**, V.S. Anil Kumar, M.V. Marathe,A. Srinivasan, Z. Toroczkai, N. Wang, *Nature* (2004) [EpiSims/TRANSIMS at LANL]





- early detection and targeted vaccination,
- stay home ("early withdrawal")

See Poster

Summary

- SW links can facilitate synchronization of parallel computing networks with many nodes
- Relevant node-to-node process is *relaxation* in the presence of *short-tailed noise* → weak *logarithmic divergence* for the typical size of the *largest fluctuations*
- Applications: dynamic phenomena in magnetic, ecological, and epidemics models

www.rpi.edu/~korniss

H. Guclu, and G.K., *Phys. Rev. E* **69**, (R) (2004).

B. Kozma, M.B. Hastings, and G.K, *Phys. Rev. Lett.* **92**, 108701 (2004).

G. K., M.A. Novotny, H. Guclu, Z. Toroczkai, and P.A. Rikvold, *Science* **299**, 677 (2003).

G. K., Z. Toroczkai, M.A. Novotny, and P.A. Rikvold, *Phys. Rev. Lett.* **84**, 1351 (2000).