Electron spin decoherence due to interaction with a nuclear spin bath

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Research Objectives

The main objectives of our program have been to explore coherent quantum mechanical processes in novel solid-state semiconductor information processing devices with components of atomic dimensions. These include quantum computers, spintronic devices, and nanometer-scale logic gates.

Approach

Our approach has been truly interdisciplinary. For example, in developing new measures of decoherence for quantum computing, we have employed concepts from many-body quantum physics, computer error-correction algorithms, and nonequilibrium statistical mechanics. In our description of spintronic devices, we have utilized large-scale Monte Carlo simulations, knowledge from solid-state physics of semiconductors and from the microelectronics area of electrical engineering, as well as novel ideas of coherent control of quantum dynamics.

Significant Results

Our achievements to date include:
- new measures of initial decoherence, and evaluation of decoherence for spins in semiconductors;
- evaluation of solid-state quantum computing designs;
- studies of transport associated with quantum measurement;
- investigation of spin-polarized devices and role of nuclear spins in spintronics and quantum computing;
- general contributions to quantum computing algorithms and to time-dependent and phase-related properties of open many-body quantum mechanical systems;
- novel analytical and numerical Monte Carlo approaches to studying spin-polarization control for spintronic device modeling;
- investigation of spin relaxation dynamics in two-dimensional semiconductor heterostructures.

Broader Impact

We have extensive research collaborations with leading experimental and theoretical groups. The educational impact has included training undergraduate and graduate students, postdoctoral researchers, and development of three new courses to introduce quantum device and quantum algorithmic concepts to graduate and undergraduate students. Our program has contributed to homeland security and received funding from the National Security Agency.

Our outreach program has included sponsoring presentation events, an international workshop series Quantum Device Technology, held in May of 2002 and May 2004, and sponsored by the Nanotechnology Council of IEEE and NSA (via ARO). We have worked with the REU site for students at SUNY Potsdam to guide several undergraduate research projects in the topics of quantum computing and quantum algorithms.
Collaboration

Experiment:

Sean Barrett, Yale
Hong Wen Jiang, UCLA
Marco Fanciulli, MDM Laboratory, Milan, Italy

Theory:

Leonid Fedichkin, Clarkson University
Boris Malkin, Kazan State University, Russia
Dima Mozyrsky, LANL
Vladimir Privman, Clarkson University
Israel Vagner, Holon Institute of technology, Israel
Donor electron spin in Si:P

Structure

Natural Silicon:

$^{28}\text{Si} - 92\%$

$^{29}\text{Si} - 4.7\%$ I=1/2

$^{30}\text{Si} - 3.1\%$

Natural Phosphorus:

$^{31}\text{P} - 100\%$ I=1/2

In the effective mass approximation the electron wave function is $s$-like:

$$F(r) = \frac{1}{\sqrt{\pi ab}} e^{-\sqrt{(x^2+y^2)/a^2+z^2/b^2}}$$
Donor electron spin in Si:P

An application for QC


$^{31}$P donor
Qubit – nuclear spin
Qubit-qubit interaction – electron spin


$^{31}$P donor
Qubit – electron spin
Qubit-qubit interaction – electron spin

Bohr Radius:
Si: $a \approx 25$ Å 
$b \approx 15$ Å
Ge: $a \approx 64$ Å 
$b \approx 24$ Å
Donor electron spin in Si:P
Sources of decoherence

• Interaction with phonons
  D. Mozyrsky, Sh. Kogan, V. N. Gorshkov, G. P. Berman

• Gate errors
  X. Hu, S. Das Sarma, cond-mat/0207457

• Interaction with $^{29}$Si nuclear spins
  Theory

  Experiments
**Donor electron spin in Si:P**

**Spin Hamiltonian**

\[
H_{\text{Spin}} = H^e_Z + \sum_i H^\text{nucl}_Z (i) + \sum_i H_{\text{Hf}}(i) + \sum_{i \neq j} H_{\text{Dip}}(i, j)
\]

- **Effect of external field**
- **Electron-nuclei interaction**
- **Nuclei-nuclei interaction**

**Electron spin Zeeman term:**
\[
H^e_Z = g \beta HS
\]

**Nuclear spin Zeeman term:**
\[
H^\text{nucl}_Z(i) = -\gamma_i \hbar H I
\]

**Hyperfine electron-nuclear spin interaction:**
\[
H_{\text{Hf}}(i) = S A_i I^i
\]

**Dipole-dipole nuclear spin interaction:**
\[
H_{\text{Dip}}(i, j) = I^i D_{ij} I^j
\]

- Effective Bohr radius $\sim 20-25$ Å
- Lattice constant = 5.43 Å
- In a natural Si crystal the donor electron interacts with $\sim 80$ nuclei of $^{29}\text{Si}$
- System of $^{29}\text{Si}$ nuclear spins can be considered as a spin bath
Donor electron spin in Si:P

Hyperfine interaction

Contact interaction:

\[ H_{\text{Cont}} = A \mathbf{S} \mathbf{I} \]

Dipole-dipole interaction:

\[ H_{\text{Dip}} = \frac{\mathbf{\mu}_e \mathbf{\mu}_n}{r^3} - \frac{3(\mathbf{\mu}_e \mathbf{r})(\mathbf{\mu}_n \mathbf{r})}{r^5} \]

Hyperfine interaction:

\[ H_{\text{Hf}} = \begin{pmatrix} S_x & S_y & S_z \end{pmatrix} \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix} \begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix} \]

Approximations:

Contact interaction only:

\[ \mathbf{A} = \begin{pmatrix} A_{xx} & 0 & 0 \\ 0 & A_{yy} & 0 \\ 0 & 0 & A_{zz} \end{pmatrix} \]

High magnetic field

\[ \mathbf{A} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix} \]

Contact interaction

High magnetic field

\[ \mathbf{A} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_{zz} \end{pmatrix} \]
Donor electron spin in Si:P

Energy level structure (high magnetic field)

\[ H_Z^e = g \beta H z S_z \]

\[ H_Z^p = -\gamma_p \hbar H z I_z^p \]

\[ H_{Hf}^p = A_{zz}^p S_z I_z^p \]

\[ H_{Si}^{Si} \]

\[ H_{Hf}^{Si} \]

\[ \hbar \]

- \(^{31}\)P electron spin

- \(^{31}\)P nuclear spin

- \(^{29}\)Si nuclear spin
Donor electron spin in Si:P

Effects of nuclear spin bath (low field)

\[ H_Z^e \sim H_z \]

\[ \Delta H^p = \text{const} \approx 42 \text{Oe} \]

\[ \delta_{\text{Si}} = \text{const} \approx 3 \text{Oe} \]

\[ 1/T [\mu s] = 1 - e^{-t/T} \]

Donor electron spin in Si:P
Effects of nuclear spin bath (high field)

(a) $S=\downarrow$

(b) $S=\uparrow$

Electron spin system

Nuclear spin system

\[ A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix} \]
**Donor electron spin in Si:P**

*Hyperfine modulations of an electron spin qubit*

Threshold value of the magnetic field for a fault tolerant 
$^{31}$P electron spin qubit:

$$\left\| \Delta \rho_{\text{max}} \right\| \sim H^{-2}$$

$$H_{\text{th}} \approx 9 \text{T}$$

Donor electron spin in Si:P

Spin echo modulations: Experiment

Spin echo:

E. Abe, K. M. Itoh, J. Isoya
S. Yamasaki, cond-mat/0402152

M. Fanciulli, P. Hofer, A. Ponti
Conclusions

• Effects of a nuclear spin bath on the decoherence of an electron spin qubit in a Si:P system has been studied.

• A new measure of decoherence processes has been applied.

• In the low field regime the coherence of a qubit decays exponentially with a characteristic time, $T \sim 0.1 \, \mu\text{sec}$.

• In the high magnetic field regime, quantum operations cause the qubit state to deviate from the ideal state. The characteristic time for these processes is on the order of $0.1 \, \mu\text{sec}$.

• The threshold value of an external magnetic field required for fault-tolerant quantum computation is $H_{\text{ext}} \sim 9$ Tesla.

Future prospects

• **Spin diffusion**

A. M. Tyryshkin, S. A. Lyon, A. V. Astashkin, and A. M. Raitsimring

• **Initial drop of spin coherence**

M. Fanciulli, P. Hofer, A. Ponti

• **Control for spin-spin coupling in solids**

S. Barrett’s Group, Yale
M. Fanciulli Group, MDM Laboratory, Italy

**Development of error avoiding methods for spin qubits in solids.**